EXPLORING TESSELLATIONS

Background: What is a tessellation?

A *tessellation* is any pattern made of repeating shapes that covers a surface completely without overlapping or leaving any gaps. A checkerboard is a tessellation made of squares. The squares meet edge to edge with no gaps and no overlapping areas. The pattern of bricks on a wall is a tessellation made of rectangles.

Over 2,200 years ago, ancient Greeks were decorating their homes with tessellations, making elaborate mosaics from tiny, square tiles. Early Persian and Islamic artists also created spectacular tessellating designs. More recently, the Dutch artist M. C. Escher used tessellation to create enchanting patterns of interlocking creatures, such as birds and fish.

Making tessellations combines the creativity of an art project with the challenge of solving a puzzle.

Part One: Tessellating with One Shape

[10 minutes]

Materials:

- Pattern blocks (multiple sets)
  You can purchase these online, or download and print out paper patterns here: http://mason.gmu.edu/~mmankus/Handson/manipulatives.htm
- Index card 3"x 5"

Try This:

Step 1   Choose one pattern block shape. Do you think many blocks of this shape will cover your card, leaving no spaces? Try it. How many shapes did it take to cover the card? How many fit across? How many fit up-and-down?

Step 2   Check out other people's work—did their shapes cover the card? It is possible to do this with every pattern block shape. The process is called tessellating.

## Part Two: Tessellating with Two Shapes

[30 minutes]

Materials:

- Pattern blocks (multiple sets), excluding the orange squares and white rhombuses
- Triangular Grid Paper (included)
- Markers
- Plain paper
- Pencils
- Tape

Try This:

Step 1   Now that you know you can tessellate with any one of these shapes, try choosing two pattern block shapes. Put the two blocks together to make a unit, and use a small piece of tape to hold them together. How many sides does this new shape have? If a side of the triangle = 1, what is the perimeter of the new shape?
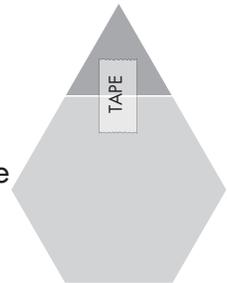
Step 2   Describe out loud how you have arranged the two shapes. (For example: "The green triangle is above the yellow hexagon. One side of the triangle matches exactly with one side of the hexagon.") Now build 10 more of the same unit, taping these together as well.

Step 3   Can you tessellate space (cover the paper leaving no spaces in between) with this unit? If not, try making a new unit with two blocks.
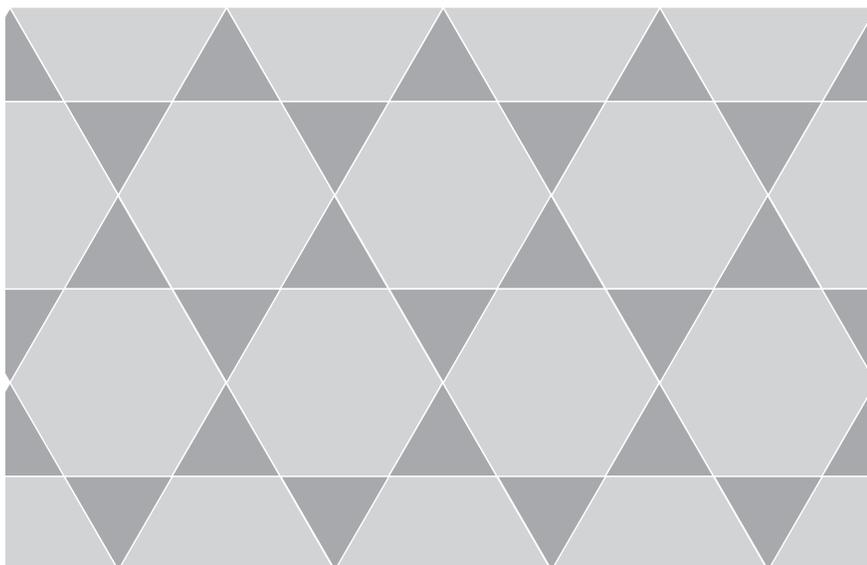
Step 4   Look at other people's work as well. Were they able to tessellate with their shapes? Describe out loud how you arranged the units to make a tessellation.

Step 5   If you placed your units side-by-side without turning or flipping them, you made a *translation tessellation*. If you had to turn your units to fit them together (like a pinwheel), you made a *rotation tessellation*. If you had to flip your units over to the other side, you made a *reflection tessellation*. You may have had to do one, two, or all three of these things to make your tessellation.

Step 6   You have made a repeating pattern, or periodic tessellation. How many ways can you arrange your two blocks into a unit to make periodic tessellations? Look at your classmates' work to see if they have tried anything that you haven't.

Step 7   Once you have made a tessellation that you like, you can preserve it by either tracing the shapes, or by drawing them onto the triangular grid paper, freehand.

## Part Three: Tessellating with Three Shapes
[45 minutes]

Materials:

- Pattern blocks (multiple sets), excluding the orange squares and white rhombuses
- Triangular Grid Paper (included)
- Markers
- Plain paper
- Pencils
- Tape

Try This:

Step 1   Now try the same exercise with three different pattern block shapes. Some units will not tessellate. Were you able to tessellate space in a repeating pattern? Was this easier or harder than with one or two shapes?
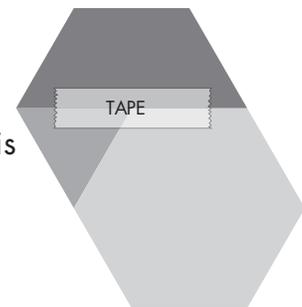
Step 2   If you placed your units side-by-side without turning or flipping them, you made a translation tessellation. If you had to turn your units to fit them together (like a pinwheel), you made a rotation tessellation. If you had to flip your units over to the other side, you made a reflection tessellation. You may have had to do one, two, or all three of these things to make your tessellation.
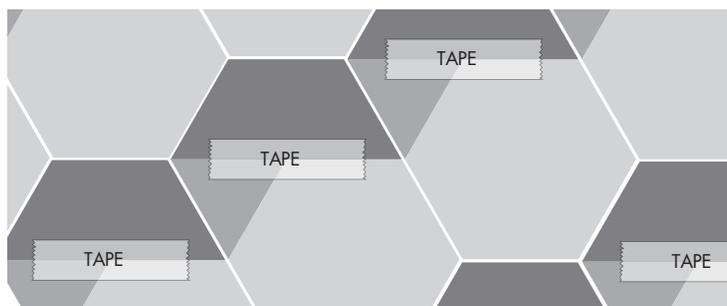
Step 3   After you've tried as many combinations of translation, rotation, and reflection as you can, don't be afraid to make a new unit. Record any units that don't tessellate so that you won't repeat them.

Step 4   Once you have made a tessellation that you like, you can preserve it by drawing it onto the triangular grid paper. To do this, you can either trace the shapes onto the paper, or draw them freehand.

Part Four: Tessellating Three-Dimensional (3D) Space
[20 minutes]

Materials:

- Cubes (sugar cubes, wooden blocks, or any cubes that are easy to obtain)
- Rectangular prisms (shoe boxes, toothpaste boxes, tissue boxes—as long as they are all the same size and shape)
- Cylinders (soda cans, paper towel tubes, soup cans, or any cylinders that are all the same size and shape)
- Spheres (marbles, tennis balls, or any spheres that are all the same size)
- Unsharpened pencils (If you rubber band a bunch of pencils together and look at them from the end, you will see a tessellating honeycomb pattern.)
- Any other interesting three-dimensional objects

Try This:

Step 1   Explore tessellating with three-dimensional objects and notice the similarities to and differences from working with two-dimensional objects.

Step 2   Start with the cubes. Can you stack the cubes together to fill three-dimensional space without leaving any gaps? To test this, see if there are any spaces between the cubes large enough to fit a pencil. If not, you have tessellated three-dimensional space.
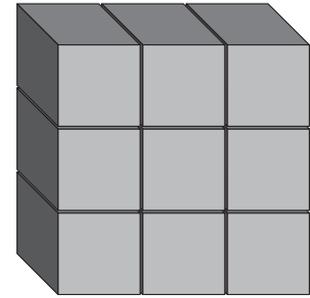
Step 3   Now try each of the other shapes. Which ones tessellate space? Which ones do not?

Step 4   What other shapes can you think of, either from nature or from your everyday life, that tessellate space? Can you think of shapes that would not tessellate space?
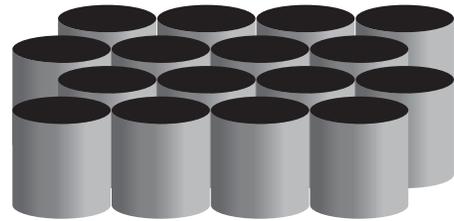
What's Going On?

Why do you think that certain objects—such as cereal boxes or honeycomb cells—tessellate space? By tessellating a space, you fill it up in the most efficient way. For example, if you can use tessellation to pack boxes into a truck or onto a market shelf, you'll be able to pack in as many boxes as possible. Likewise, honeybees use all of the available space in their hive by building their honeycomb in a tessellated pattern.
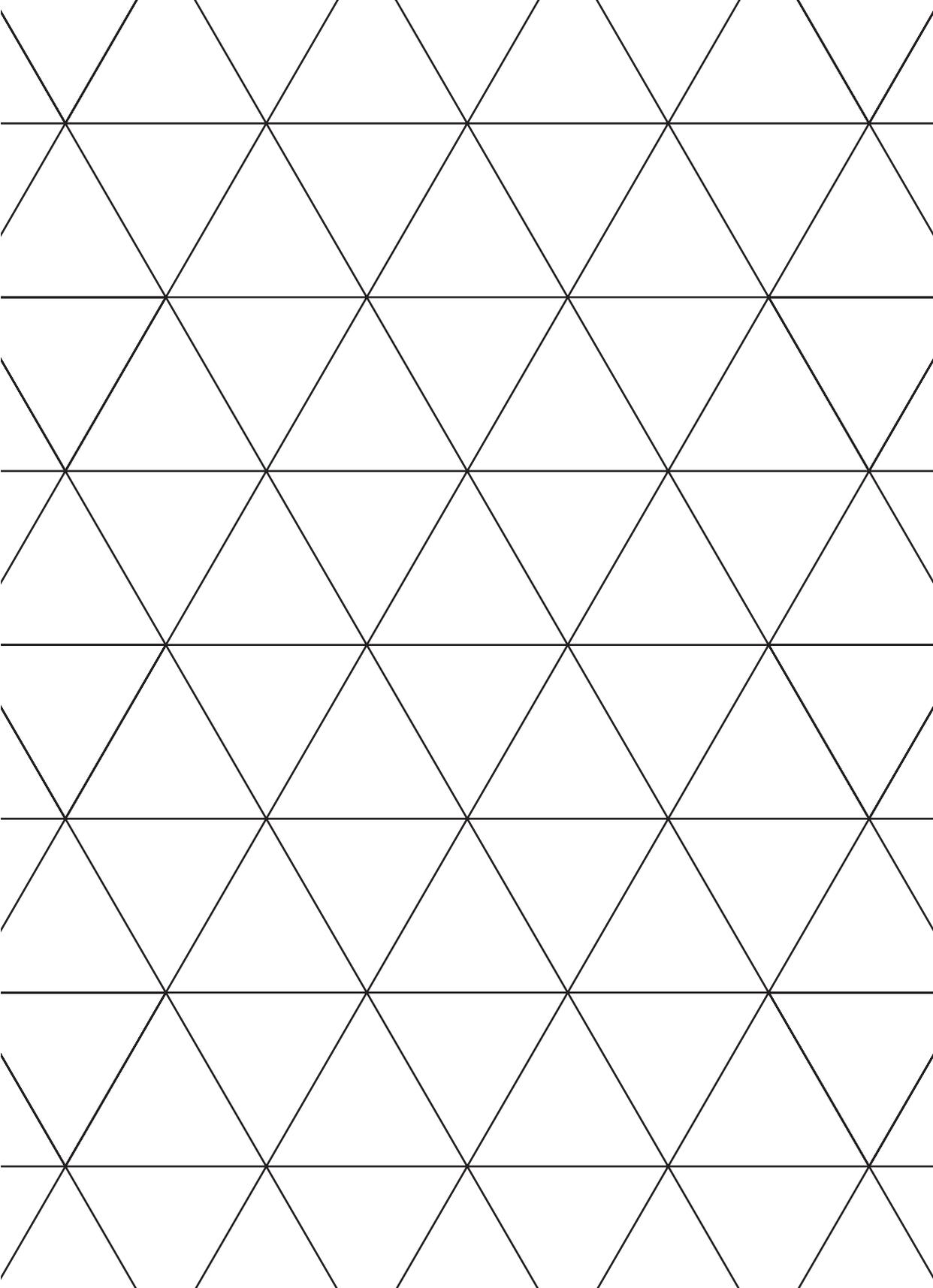
From the National Council of Teachers of Mathematics (NCTM)

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Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships:

- Investigate, describe, and reason about the results of subdividing, combining, and transforming shapes.

Apply transformations and use symmetry to analyze mathematical situations:

- Predict and describe the results of sliding, flipping, and turning two-dimensional shapes.

Use visualization, spatial reasoning, and geometric modeling to solve problems:

- Recognize geometric ideas and relationships and apply them to other disciplines and to problems that arise in the classroom or in everyday life.

Apply appropriate techniques, tools, and formulas to determine measurements:

- Develop strategies for estimating the perimeters, areas, and volumes of irregular shapes.