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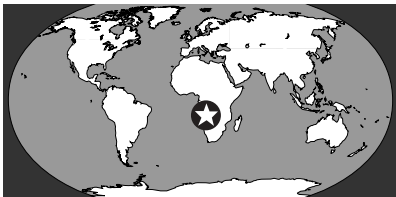


# SONA

Sand Drawings from Africa

# SONA Sand Drawings from Africa

In an area of southwestern Africa, there's a tradition of storytelling using sand drawings called *sona*, whose complex, interwoven patterns reveal some interesting mathematical ideas.



## The Sand Drawings of the Chokwe

The Chokwe (pronounced “chock way”) people live in southwestern Africa, mainly in northeast Angola. They are famous for their decorative arts, including beautiful woven mats and baskets, pottery and wood sculptures, and the striking geometric designs they use to decorate the walls of their homes.

The Chokwe also use their art in storytelling. They have an ancient tradition of making sand drawings, known in their language as *lusona* (plural: *sona*), to illustrate their stories. The *sona* illustrate proverbs, fables, games, riddles, and stories about animals.

In all cultures, there are stories that have been passed down through many generations, from the old to the young. Among the Chokwe, expert storytellers not only tell the tales well, but they also create traditional *sona* drawings to go with each story. These drawings have specific rules. The storyteller starts by making a series of dots, evenly spaced, in a rectangular array. Each drawing has a set number of rows and columns of dots.

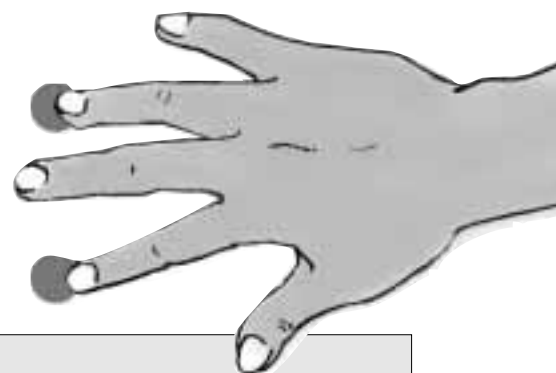


Once the narrator makes the dots, the drawing can begin.

The drawing itself is made up of lines that weave in and out around the dots. The narrator must draw and talk at the same time, without stopping. The listeners follow the storyteller closely, listening and watching as the drawing develops. If the storyteller hesitates, the audience smiles or laughs at the mistake.

In the past, the *sona* sand drawings and the stories they illustrated

played an important role in passing knowledge from one generation of Chokwe people to the next. But several hundred years of colonization and slavery have weakened the tradition of making sand drawings. Many of the drawings are no longer made today. A lot of what we know about them comes from the records of missionaries, who wrote about the *sona* and copied some of the drawings on paper.



## WHAT'S IT ALL ABOUT?

As you learn about the beautiful art of the Chokwe sona, you'll also explore the mathematical principles involved in creating these drawings. Along the way, you'll discover:

- How many unbroken lines it takes to make each sand drawing
- What algorithms are and what they have to do with sona sand drawings

### Is there anything special I should know?

- This activity is recommended for ages 10 and up
- You can do this activity by yourself, or with friends

### How much time will I need?

- This activity will take 1 to 2 hours

### What materials will I need?

#### If you're working inside, you'll need:

- Graph paper
- Colored pencils

#### If you're working outside, you'll need:

- A place to make drawings in fine dirt or sand
- About 20 small stones, uncooked beans, or other small place markers

#### You'll also need:

- Copies of "The Stork and the Leopard" pattern, on page 9 (optional)

## Making Sona Drawings

A simple sona drawing begins with dots arranged in a rectangular array. If you're using graph paper, mark the intersections of the lines on the paper with pencil dots. If you're outside, use the techniques that the Chokwe storyteller would use to make this array.

After cleaning and smoothing the sandy ground, the Chokwe storyteller first marks points with the tips of his extended fingers. He uses the index and ring fingers of his right hand to mark the points. His goal

is to make an array of points that are evenly spaced, like the intersections on your graph paper.

To mark points from right to left, the storyteller keeps the tip of his ring finger on the last point he marked on the ground, and marks a new point with his index finger. He then moves his ring finger to the new point and marks another new point with his index finger. This method guarantees that the distance between points remains the same. When the storyteller marks points from left to right, he uses his

ring finger to first mark the new points. To mark points going up or down, he simply turns his hand 90 degrees.

The storyteller now has an array of evenly spaced dots set up in a rectangular pattern. The size of the array depends on the story he is going to tell.

If you're outdoors, you can make the array with your fingers, then use small stones at each point to make the array easy to see. Try making your own array of three rows and three columns of dots. 🖐️



## Drawing a Sona Pattern

Once the points have been marked, the storyteller begins to draw, usually with the index finger of his right hand.

The drawings, which are a sort of language made up of points and lines, obey certain rules. They are always made with the fewest possible lines. Some drawings that may look very complicated can be made with only one line. The storyteller starts and finishes without ever having to lift his finger.

No matter how big the array is, or how complicated the pattern may look, both the storyteller and the

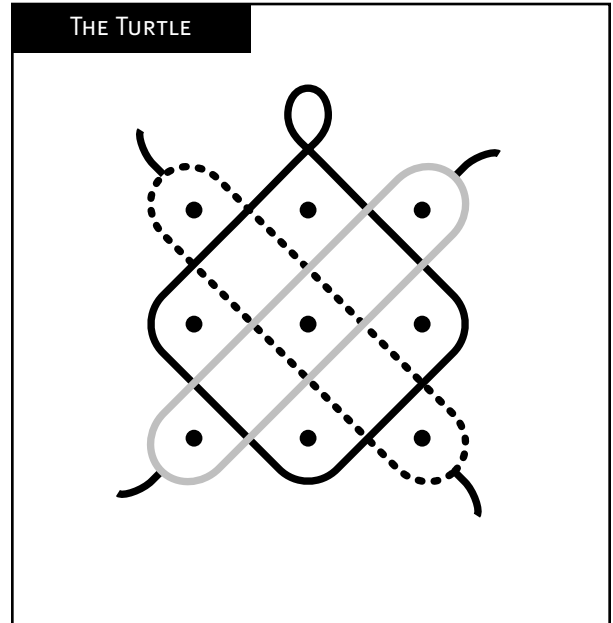
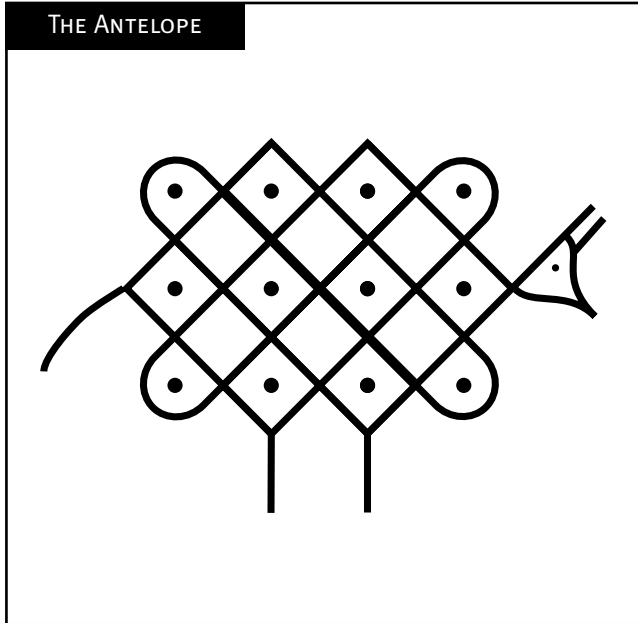
Chokwe people watching know how many lines will be needed as soon as they see the array of dots. As you re-create some sona patterns, you'll begin to discover the secret of this understanding for yourself.

Before you try to draw a sona pattern of your own, you should know five rules:

1. DO NOT connect the dots! Sona lines go around and between the dots, but never touch them.
2. For most sona drawings, you can begin your first line between any two dots of the array.
3. Once you've picked a starting point, draw a straight line at 45

degrees between the dots. When you reach the edge of the array, turn your line 90 degrees and make another straight line.

4. You may cross a line you've already made, but don't trace over the same line twice.
5. If the end of a line meets its own beginning, that's a closed line. Once you've made a closed line, you may need to find another place in the array to begin another line. (If you're using colored pencils, use a different color for each closed line to help you see the patterns more easily.)



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HOW TO DRAW THE ANTELOPE SONA.  
 THE HEAD, LEGS, AND TAIL ARE  
 ADDED AT THE END.

**The Antelope**

The sona pattern that depicts an antelope is drawn on a 3 x 4 array of dots. Make the array first and then make the drawing.

Remember that the lines go through the array diagonally. What is the smallest number of closed lines you can use to make the body of the antelope—not counting the head, legs, and tail? 🖐️

You can actually make this drawing with one closed line. You can start anywhere on the body of the antelope and come back to that point without lifting your finger or pencil. When your line comes back to where you started, you’ve finished the body. Then you can add the head, tail, and feet to finish the antelope.

If your drawing required more than one closed line, go back and try it again. Remember that the drawing does not connect the dots. Instead it passes and loops around them. The lines end up making shapes with the dots in the center.

Notice that the lines you drew to make the animal are oriented at 45 degrees, diagonally to the array. If

you’re working with graph paper, you can use your pencil to draw a very faint rectangle around the outside of the finished drawing. Wherever you start drawing, the line should go straight until it reaches the edge of this invisible rectangle. When your line reaches an edge, it must make a 90-degree turn. At the corners, you have to make a U-turn—a 180-degree turn. 🖐️

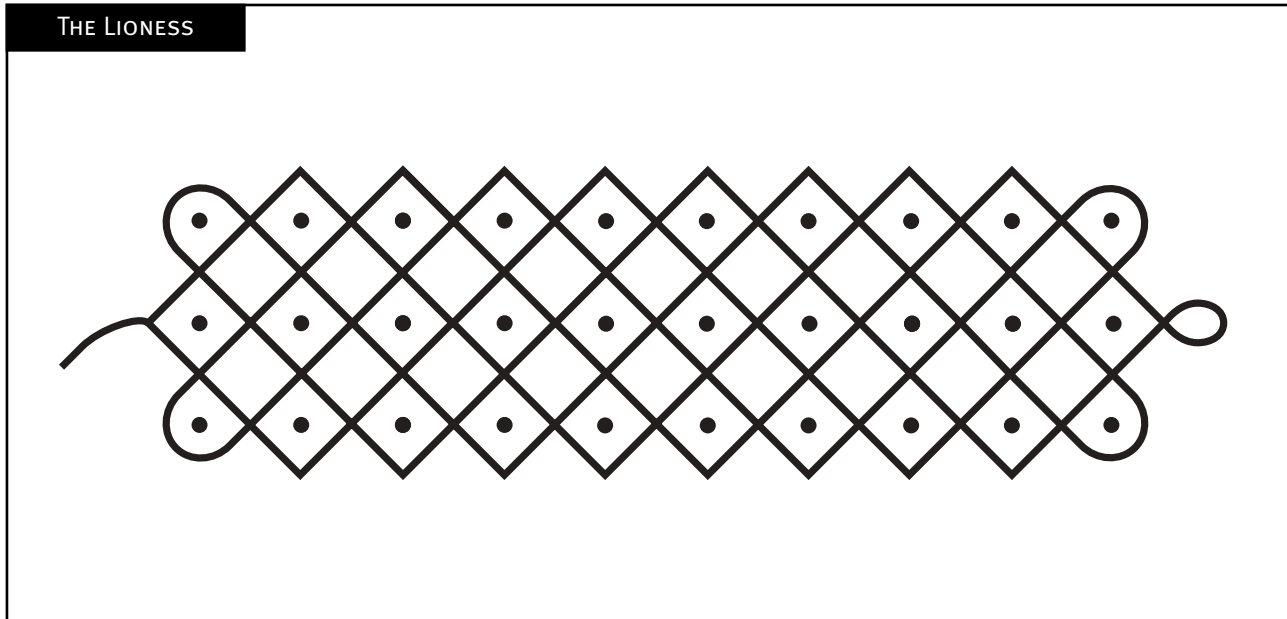
**The Turtle**

Take a careful look at the drawing of the turtle, above. You can see the points of a 3 x 3 array that the storyteller made before starting the drawing.

To draw the turtle, start by making your own 3 x 3 array of points. 🖐️

The turtle is drawn on a smaller array than the antelope. How many closed lines do you think you’ll need to draw it—not counting the four short legs?

You may be surprised to find that it takes three closed lines to draw the turtle. Each is shown as a different shade or pattern in the picture at the top of the page.



**The Lioness**

First make a 3 x 10 array. Then take a minute to study the picture of the lioness, above.

Now try re-creating the picture, using the same technique you used for the antelope. Start anywhere and draw a straight diagonal line, turning only when you reach a corner or an edge. 🖐

You’ve seen that the lines of the sona are drawn at a 45-degree angle to the sides of the invisible rectangle. What other patterns or rules do you notice as you draw the lioness? 🖐

The body of the lioness can be drawn with a single closed line. Just as in sona drawings of the turtle and the antelope, when a line reaches a side of the rectangle, it makes a 90-degree turn. When a line reaches a corner, it makes a rounded U-turn and heads back along a path parallel to the one it came in on. The head and tail are added at the end.

**Sona Physics**

There’s a physics principle hiding in these drawings. Look carefully at the two drawings at the top of page 9.

They are the basic lioness and turtle but the corners are squared off and the invisible rectangle around each drawing is thinly marked.

All the bouncing lines may remind you of billiard balls moving on a pool table or light rays hitting a mirror: When they strike a surface at an angle of 45 degrees, they bounce off at the same angle. Notice that the lines in these drawings follow the same kind of trajectory, traveling straight until they reach the edge and then bouncing off at an angle equal to the incoming angle. Once we draw the invisible rectangles around the drawings, the set of rules we used initially become equivalent to the physical laws of reflection on a surface.

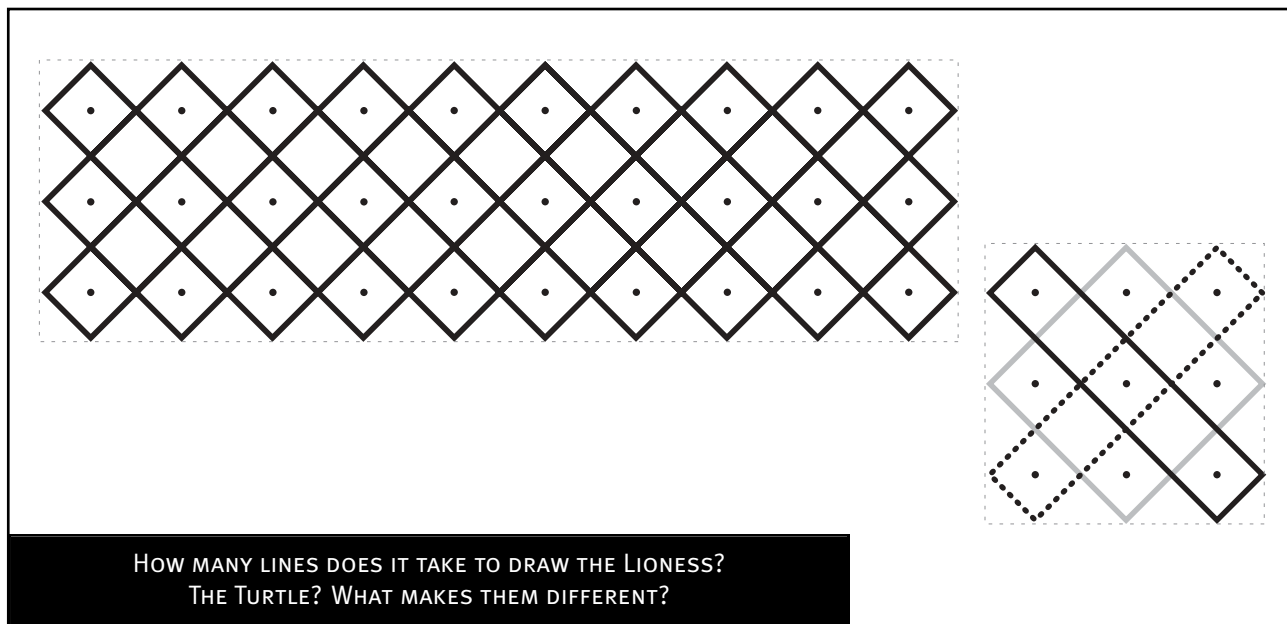
**Rules Make Rhythm**

The sequence of the movements that you make as you draw the lioness, the turtle, and the antelope follow rules or equivalent physical laws. These rules determine a *rhythm*, specific repetitions that are independent of the size of the rectangular array of points and of the number of lines necessary to com-

plete the drawing. The rhythm used to draw these three sona is different, for instance, from the one you’d need to use if you were drawing the Large Lion’s Stomach shown on page 13.

In a sense, the rhythm generated by the repeating motions you make to complete the drawing is similar to the repeated use of a mathematical operation. This is called *executing an algorithm*. An algorithm is a set of operations (like “add two, then divide by ten”) that gets you from some starting situation to a result. A key aspect of algorithms is that they always apply the same rules in any situation, but give different answers depending on the starting conditions. In our example, if you start with 18 and apply the algorithm “add two, then divide by ten,” you get 2. If you start with a different number, you get a different result.

Although all these sona designs followed the same rhythm, the number of closed lines you needed differed, depending on the number of dots in the rows and columns of the array you started with.



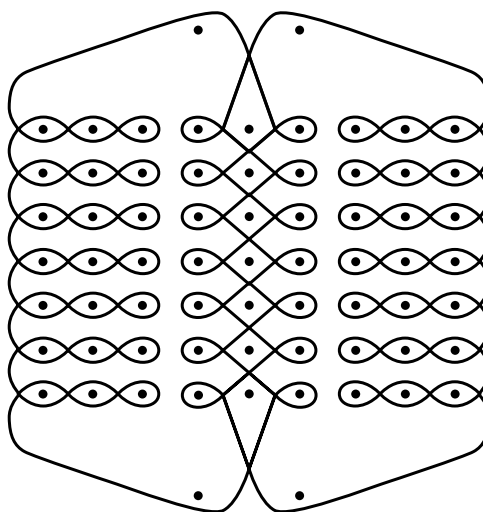
## How Many Lines?

You may have been surprised to find that the lioness and the antelope could be drawn with only one closed line, while the smaller turtle required three. The number of closed lines you need to make each drawing depends on the dimensions of the array of points you start with. For example, you need only one closed line to draw a baby lion with a  $3 \times 5$  array. Yet if you try drawing a larger lioness with a  $3 \times 9$  array, you'll find that you need three different closed lines to complete your drawing.

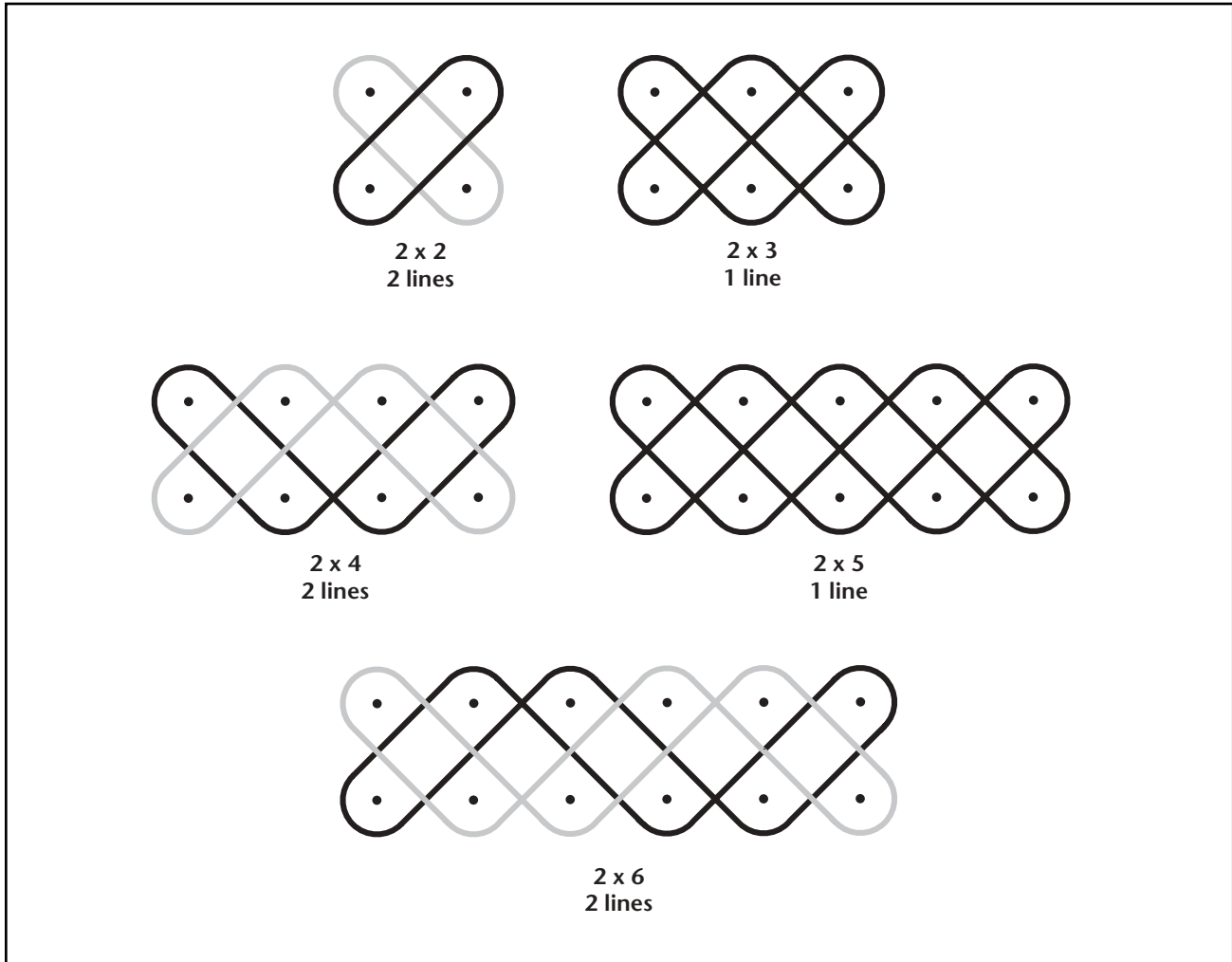
In Angola, Ghana, Congo, and other African countries, many adults and children know how many closed lines are needed to make a sona drawing with a  $4 \times 6$  array, they can tell you right away that two lines are needed. Show them a  $5 \times 7$  array, and they quickly say that one closed line will be enough.

## The Stork and the Leopard

Once the leopard Kajama asked the stork Kumbi for some feathers to line his den. Some time later, the stork asked the leopard for a piece of his skin. When Kajama granted the stork's request, he died. Kajama's sons tried to take revenge, but Kumbi, who knew the swamp very well, was able to escape.



IN THIS DRAWING, THE WINDING LINE IS THE PATH OF THE FLEEING STORK, KUMBI. THE POINTS REPRESENT THE SWAMP THROUGH WHICH KUMBI MAKES HIS ESCAPE. THE DRAWING ACTUALLY CONSISTS OF TWO INTERTWINING CURVED LINES. YOU MAY WANT TO TRACE OVER THE ESCAPE ROUTE BY FOLLOWING IT WITH TWO DIFFERENT COLORED PENCILS ON A PHOTOCOPY OF THIS DRAWING.



How can you look at an array of dots and tell how many closed lines will be needed for the drawing? To explore this question, you'll have to do some experimenting. Start with arrays based on two rows. Make arrays of  $2 \times 2$  dots,  $2 \times 3$  dots,  $2 \times 4$  dots,  $2 \times 5$  dots, and  $2 \times 6$  dots. Then follow the rules to make a drawing for each array. What do you notice? 🖐️

It turns out that arrays of two rows with an even number of dots in each row (two, four, six) need two closed lines. But when there's an odd number of dots in each row (three, five, seven), the drawings need only one closed line.

How many closed lines do you think you'll need for a  $2 \times 8$  array?

How about a  $2 \times 7$  array? Test your ideas by doing the drawings. 🖐️

You should discover that you need two closed lines for the  $2 \times 8$  array and one for the  $2 \times 7$  array.

Now take some time to explore arrays with three rows. What kind of pattern or patterns can you find for them? 🖐️

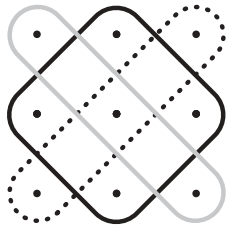
The pattern is different here. If you have three rows, you need three closed lines to draw the figure if there are three or six points in each row. But if there are four or five points in each row, only one closed line is needed. Now do some exploring with arrays that have four rows. 🖐️

The pattern is different once again. In the  $4 \times 3$  and  $4 \times 5$  arrays,

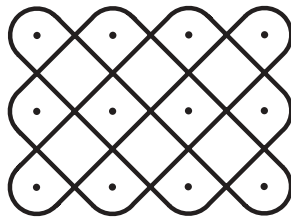
one closed line is enough. In  $4 \times 2$  and  $4 \times 6$  arrays, two lines are needed. In the  $4 \times 4$  array, you need four closed lines.

It seems as if a different rule applies for each size of array. But that's not the case. There are many patterns, but one way to predict the number of closed lines needed to draw a pattern.

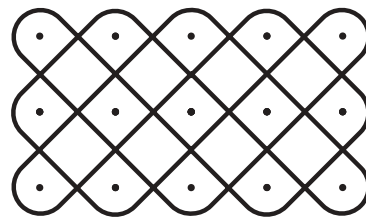
To solve the problem, think about what you already know. For each array, you start with two numbers—the number of rows and the number of columns. From these two numbers, you want to find a third number: the number of closed lines needed to do the drawing. How can you figure this out? 🖐️



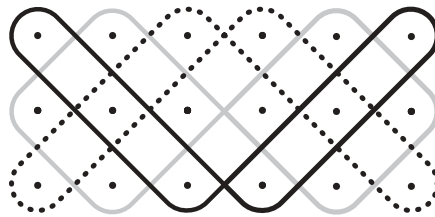
3 × 3  
3 lines



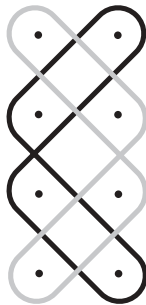
3 × 4  
1 line



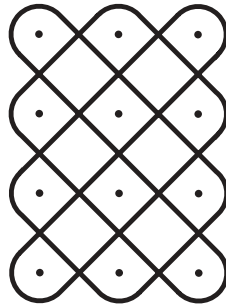
3 × 5  
1 line



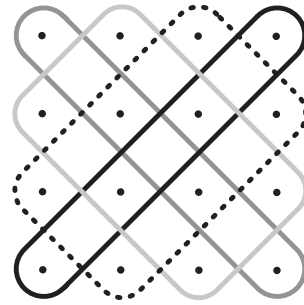
3 × 6  
3 lines



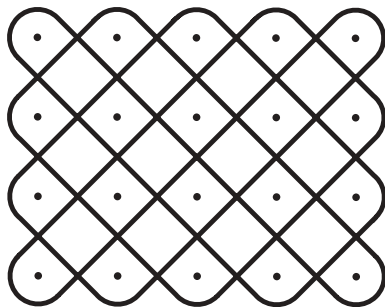
4 × 2  
2 lines



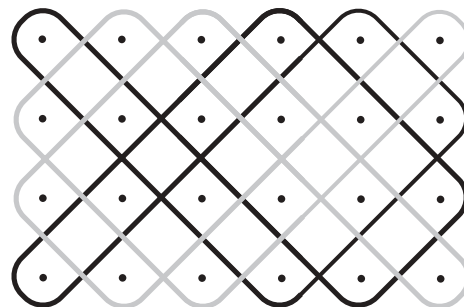
4 × 3  
1 line



4 × 4  
4 lines



4 × 5  
1 line



4 × 6  
2 lines

HOW MANY CLOSED LINES DO YOU NEED TO COMPLETE EACH PATTERN?  
IT DEPENDS ON THE NUMBER OF ROWS AND COLUMNS IN EACH ARRAY.

Number of Rows	Number of Columns	Number of Closed Lines
<b>2</b>	<b>2</b>	<b>2</b>
<b>1</b>	<b>3</b>	<b>1</b>

**CAN YOU FIGURE OUT HOW THE NUMBER OF CLOSED LINES YOU NEED TO DRAW A LUSONA IS RELATED TO THE NUMBER OF ROWS AND COLUMNS IN THE ARRAY?**

In mathematics, there are many ways to get one number from two others. For example, you could add the first two numbers or subtract one from the other; you could multiply or divide the numbers; you could find the lowest common multiple of the two. You can probably think of many other ways to get a number from two other numbers.

When you have several groups of three numbers and you're trying to figure out the rule that connects them, it's often helpful to make a table. Try making a table for the arrays you've worked with. Set up your table with three columns: Number of Rows, Number of Columns, and Number of Closed Lines, as shown above.

Now see if you can find a mathematical relationship between the numbers in the third column of your

table and the numbers in the first and second columns. You're looking for a relationship, or rule, that is true for every row in your table. Is the third number the sum of the two others? Is it the difference between them? Or is the rule more complicated?

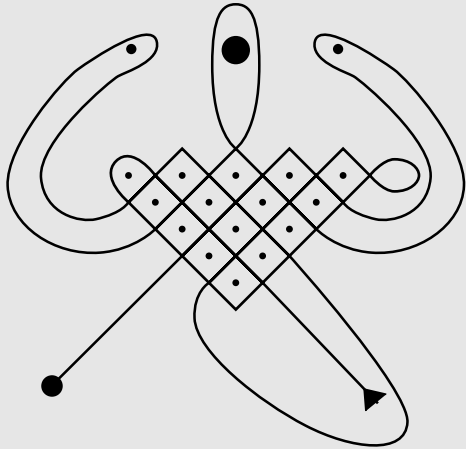
If after a while you don't see a rule, don't give up. A lot of work in math and science involves exploring a set of data until you see a pattern that was there all along. Keep trying! Your work will be rewarded with the "Aha!" of discovery.

Did you discover the pattern? Here it is: The third number is the largest number that divides evenly into the first two numbers. That is, the number of closed lines is equal to the greatest common divisor of the number of rows and the number of columns in the array.

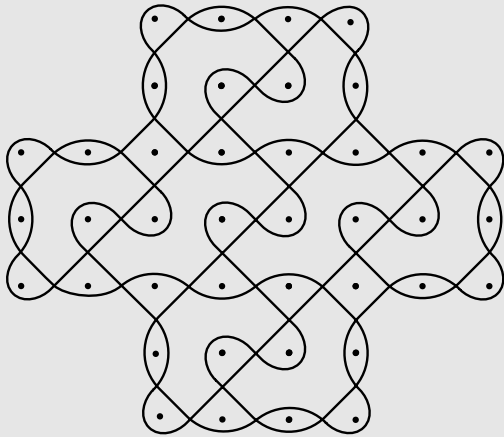
For example, the greatest common divisor of 3 and 6 is 3, so it takes three closed lines to fill a 3 x 6 array. The greatest common divisor of 3 and 5 is 1, so you know you need just one closed line to complete a 3 x 5 drawing. What is the greatest common divisor of 4 and 2? Does that fit with the results on your table?

The greatest common divisor of 4 and 2 is 2—and it takes two lines to complete a 2 x 4 array.

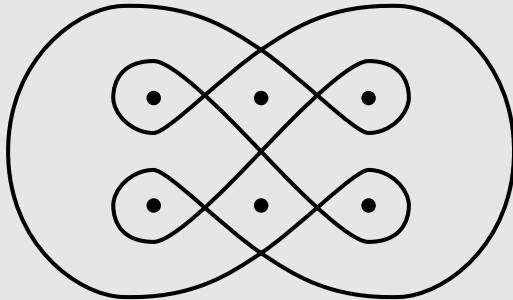
Look again at your table to be sure the rule holds true for all the arrays. You can test the rule by making other drawings with arrays you haven't tried yet. For example, how many lines should it take to draw a 3 x 12 lioness? Try it and see!



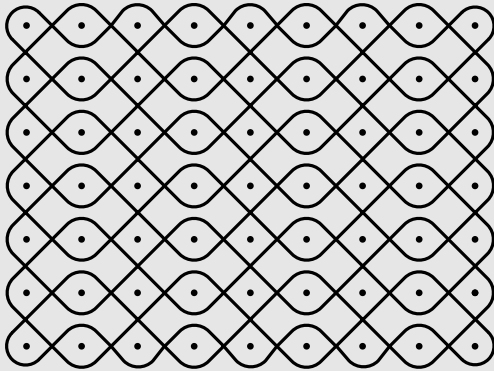
FLYING BIRD



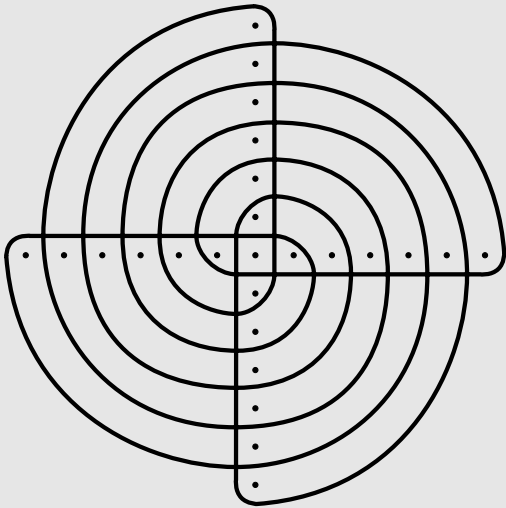
CHASED CHICKEN'S PATH



ANTELOPE FOOTPRINT



LARGE LION'S STOMACH



SPIDER IN ITS WEB

**More Sand Drawings**

These symmetrical Chokwe drawings are more complex than the rectangular arrays you have been working with. Each style—the Chased Chicken's Path, the Antelope Footprint, the Flying Bird, the Spider in Its Web, and the Large Lion's Stomach—has different rules of its own that must be followed by the storyteller. Enjoy these other styles of sona drawings for their intricate beauty, or explore further and see if you can re-create any of them.

**This activity was developed  
by Paulus Gerdes.**

## **Making Connections**

- Does your family keep any photo albums? Are there family stories that go with the pictures? How did you learn those stories? Think about what might happen if one of your descendants looked at those albums a hundred years from now. Do you think they will know those stories, too? How?
- You probably learned many fairy tales when you were young. Ask people who come from different parts of the world about the stories they learned when they were young. Are they the same kinds of stories as yours?
- Many stories are told to help people learn a lesson or understand a problem. What have you learned from the stories, proverbs, and fables you have been told?

## **Recommended Resources**

Frankenstein, Marilyn, and Arthur Powell, eds. *Ethnomathematics*. New York: State University of New York Press, 1997.

Gerdes, Paulus. *Geometry from Africa: Mathematical and Educational Explorations*. Washington, DC: Mathematical Association of America, 1999.

Jordan, Manuel. *Chokwe* (Heritage Library of African Peoples/Central Africa). New York: Rosen Publishing Group, 1997.

Jordan, Manuel et al. *Chokwe: Art and Initiation Among Chokwe and Related Peoples*. Prestel USA, 1998.

Zaslavsky, Claudia. *Africa Counts: Number and Pattern in African Culture*. New York: Lawrence Hill Books (3d edition), 1999.