5th NGSS STEM Conference
HEADING INTO THE
1st DIMENSION:
Science and Engineering Practices
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Handy Measuring Tool

Measure the height of an object indirectly using your hand and a ratio.

Many of the numbers we use in science have never been measured directly; we only know them from indirect measurements. How far is it to the sun? What is the diameter of Saturn's rings? Here's a way to use nonstandard measurements and simple ratios for estimating sizes or distances.

Tools and Materials

- Your hand
- Meter stick or measuring tape
- Partner
- String
- Tape

Assembly

None needed.

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To Do and Notice

Measuring with your extended hand

Open your hand and fully extend your fingers. You will be using the distance between the tip of your thumb and the tip of your little finger as a measuring tool for finding height. (In some Spanish-speaking cultures, this is called a *cuarta*.) The height of your extended hand, from the thumb to the little finger, is 1 unit.

Measure the length of your arm using extended hands (or *cuartas*) as your measuring unit. What is the ratio of the height of your extended hand to the length of your arm measured in hands? Round it to a simple fraction. Most people are about 1 to 3, or 1:3, meaning that their arms are about 3 *cuartas* long.

Close one eye and look down your straightened arm to locate an object in the distance that you can just barely span the height of with your extended hand. The top of the object should just line up with the top of your thumb, and the bottom of the object should just line up with the tip of your little finger. At this distance, the ratio of the height of the object you've obscured to the distance between the object and your eye is the same as the hand-to-arm length ratio you determined earlier. Therefore, whatever you can obscure with your extended hand *is three times farther away from you than it is tall.*

Mark your location on the floor with a piece of tape. Use the tape as a reference, and measure your distance to the object in centimeters.

Use the known ratio and the distance to the object to find the height of the object without measuring it. This is called an *indirect measurement*.

$$\frac{\text{height of hand with fingers extended}}{\text{arm length}} = \frac{\text{object height}}{\text{distance to object}}$$

Now measure the height of the object directly with your measuring tape. How well did the Handy Measuring Tool method work?

Calculate your error: Find the difference between the height you found indirectly and the height you measured directly. Divide this difference by the actual measured height of the object. Multiply this quotient by 100 to get percent error.

What angle is swept out by your hand when your fingers are extended? (Hint: When both of your arms are outstretched at shoulder level, they sweep out a 180˚ angle.)

Measuring with your closed fist

Using the same procedure, find the ratio for your fist-to-arm length ratio. Will it be a larger or smaller ratio than the extended hand-to-arm length ratio?

Measure the length of your arm length in fists (do not include your thumb) when your arm is extended at shoulder height. Be careful not to roll your fist along your arm.

What is the ratio of height of your fist to length of your arm? Round it to a simple fraction. Most people have a 1:7 ratio.

Close one eye and look down your straightened arm to locate an object that you can *just cover* with your fist. The top of the object should just line up with the top of your fist, and the bottom of the object should just line up with bottom of your fist. At this distance, the ratio of the height of the object you've obscured to the distance between the object and your eye is the same as the fist-to-arm length ratio you determined (for most people, 1:7). Make sure you are facing the object and keeping your elbow straight. The ratio changes if you bend your arm.

Mark your location on the floor with a piece of tape. Use the tape as a reference, and measure your distance to the object in centimeters.

Use the known ratio and the distance to the object to find the height of the object without measuring it.

Measure the height of the object directly to check your work.

Calculate your error: Find the difference between the height you found indirectly and the height you actually measured. Divide this difference by the actual measured height of the object. Multiply this quotient by 100% to get percent error.

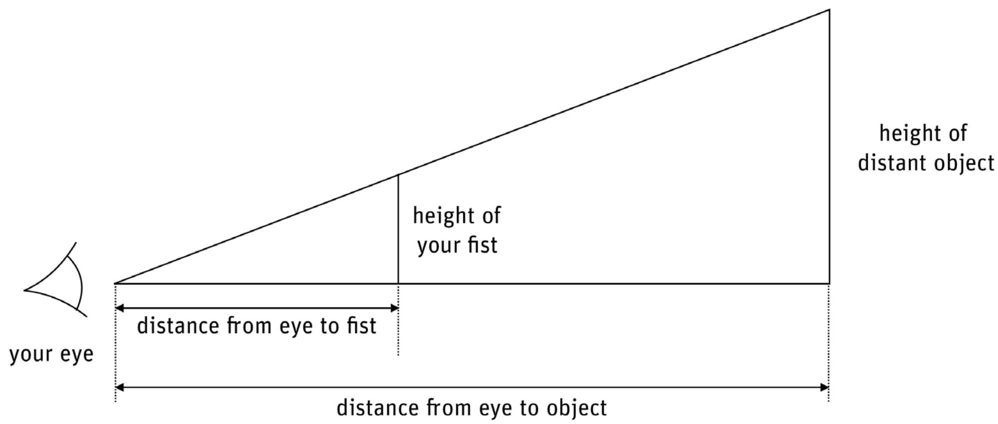
What angle is swept out by your fist? (Hint: Compare your fist to something known, like your extended hand, or to a known angle.)

What's Going On?

When you use your extended hand to just obscure the height of a distant object, that object sweeps out, or subtends, the same angle as your extended hand does on your eye's retina. Two similar triangles are created; one is embedded within the other. The embedded triangle's base is the length of your arm, and its altitude is the height of your hand with the fingers extended. The larger triangle's base is the distance to the faraway object, and its altitude is the height of the distant object. These heights and distances are proportional:

$$\frac{\text{height of hand with fingers extended}}{\text{arm length}} = \frac{\text{object height}}{\text{distance to object}}$$

A geometric proof of why this works can be shown using similar triangles.



You can model these triangles with a long piece of string. Ask a partner to extend her hand and find an object that she can just obscure with her hand when her arm is extended. Tape one end of the string to the top of the object. Bring the string toward your partner's eye, noticing how the top of her thumb and the top of the object she obscured with her hand line up. Ask her to hold the string next to her eye then continue with the rest of the string back to the bottom of the object. Pull the string taut, and you'll see a large triangle, with two sides formed by the string and one side by the object. A smaller triangle is embedded in this large triangle. It has two sides formed by the string, and its third side formed by her hand. These two triangles are similar.

You can find the angle subtended by your hand by comparing it to an easy "known" angle. When both of your arms are outstretched at shoulder level, they sweep out a 180˚ angle. How many extended hands fit in this 180˚ angle? By carefully stacking your extended hand unit from horizon to horizon, you should find that you can fit about 9 of them in the 180˚ angle. 180˚/9 = 20˚; therefore, your extended hand measures a 20˚ angle.

Going Further

This is a tool that is used by astronomers to measure objects in the sky.

When your hand is extended in front of you, and your elbow is straight, your hand obscures (or subtends) about 20˚ (assuming that your eye is the vertex of the angle) of your vision. If your extended hand subtends about 20˚ when your elbow is straight, how many degrees do you think your fist subtends when your arm is straight?

You can measure the sizes of different parts of your hand to determine the ratios and angles in the table below. For example, about two fists can fit in one extended hand. If the extended hand sweeps out, or subtends, 20˚, then your fist sweeps out about 10˚.

Here are common simple ratios for other Handy Measuring Tools:

height/distance	simple ratio	degrees subtended
height of extended hand / Length of your arm	1:3	20˚
height of fist / length of your arm	1:7	10˚
width of thumb nail / length of your arm	1:30	2˚
width of little finger nail / length of your arm	1:60	1˚

With various combinations of these angular tools, you can measure many objects in the night sky. Can you determine the angular diameter of the Big Dipper or the full moon?